



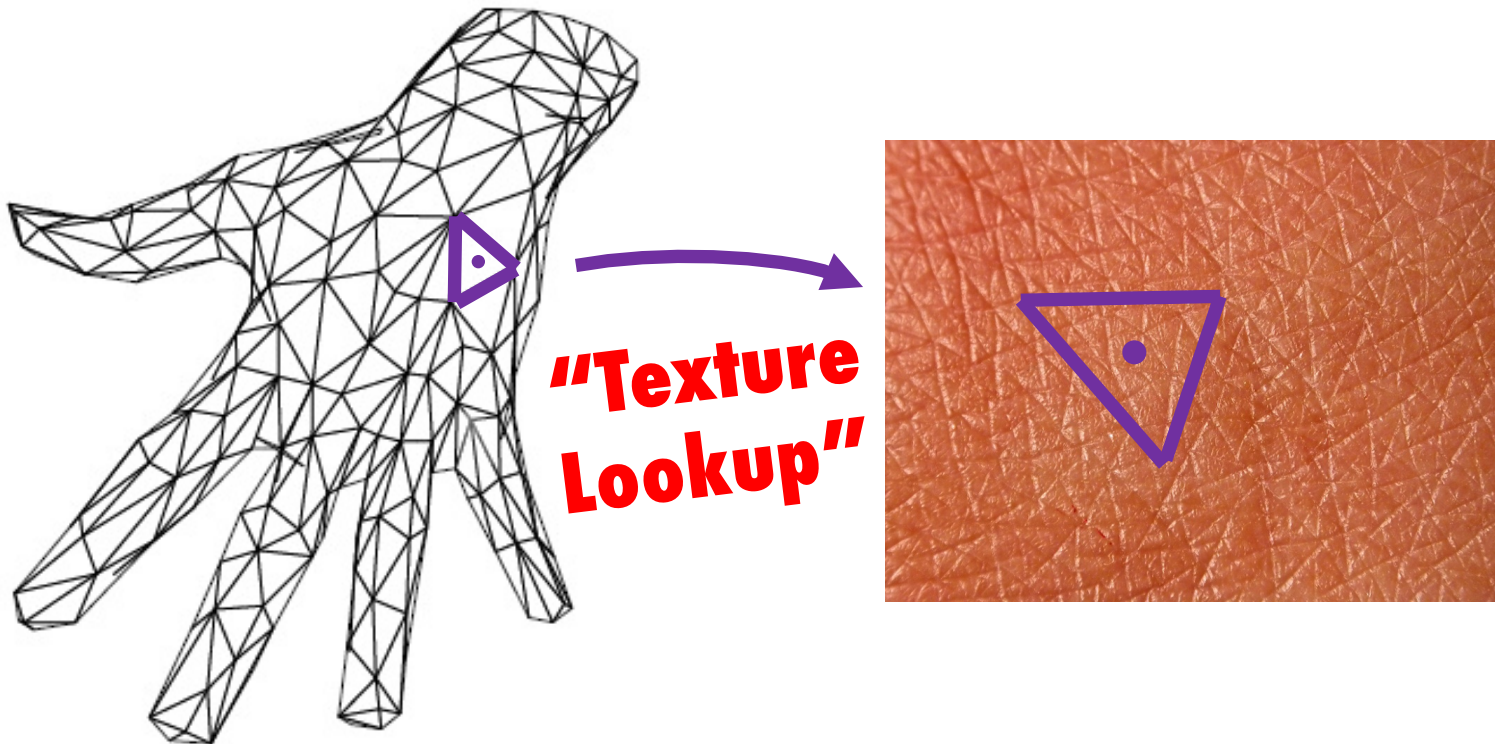
Textures



CS 148: Summer 2016
Introduction of Graphics and Imaging
Zahid Hossain

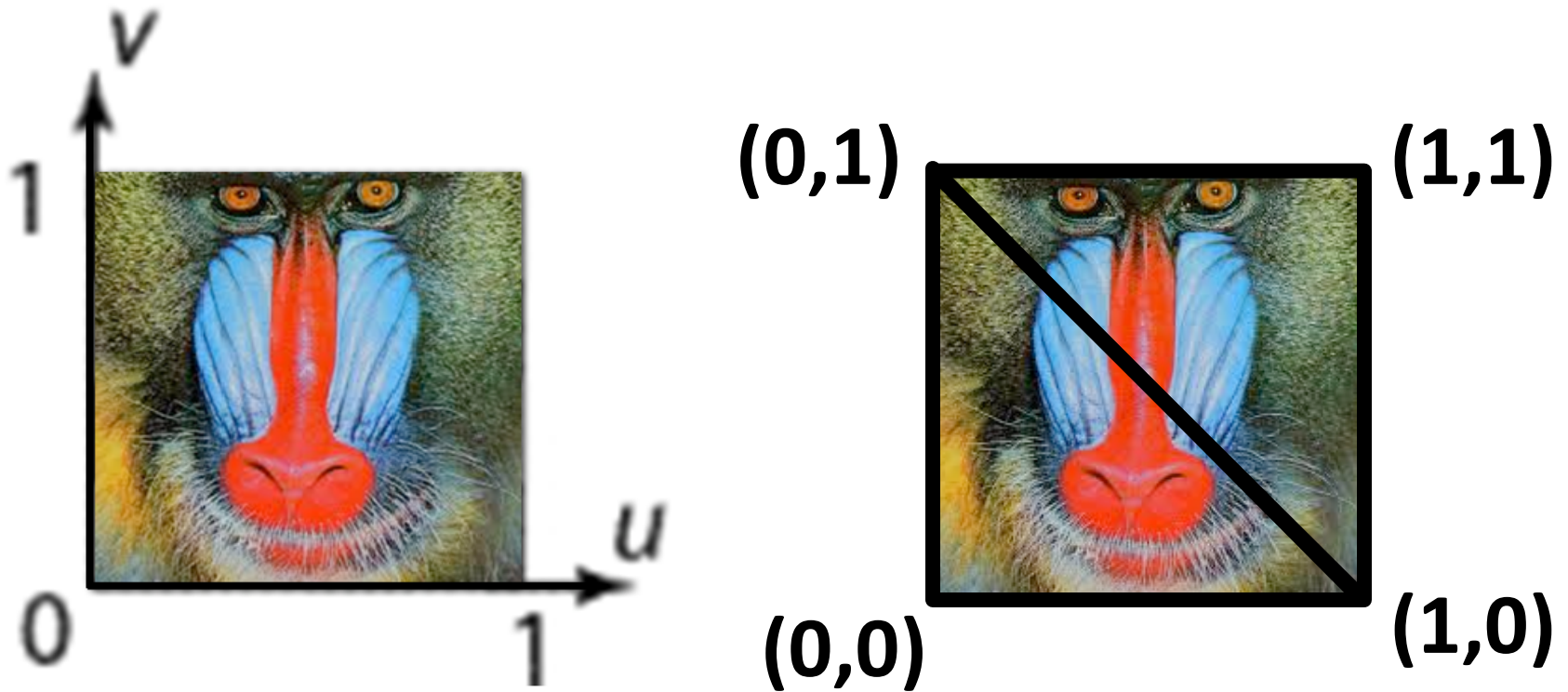
Texture Mapping

- A technique for specifying variations in surface reflectance properties of an object
- Store the reflectance as an image and “map” it onto the object
- The stored image is called a texture map



Texture Correspondence

- A texture map is defined in its own 2D coordinate system, parameterized by (u, v)
- Establish a correspondence by assigning (u, v) coordinates to triangle vertices



OpenGL Texturing Snippet

```
glEnable(GL_TEXTURE_2D);
// Create texture object
mTexId = 0;
glGenTextures(1, &mTexId);

// Create pixel data and fill it out
unsigned char *pixels = new unsigned char [width * height * 4];

// Load pixel data into OpenGL/GPU, 4 bytes/pixel, RGBA
glBindTexture(GL_TEXTURE_2D, mTexId);
glTexImage2D(GL_TEXTURE_2D, 0, GL_RGBA,
             width, height, 0,
             GL_RGBA, GL_UNSIGNED_BYTE, pixels);

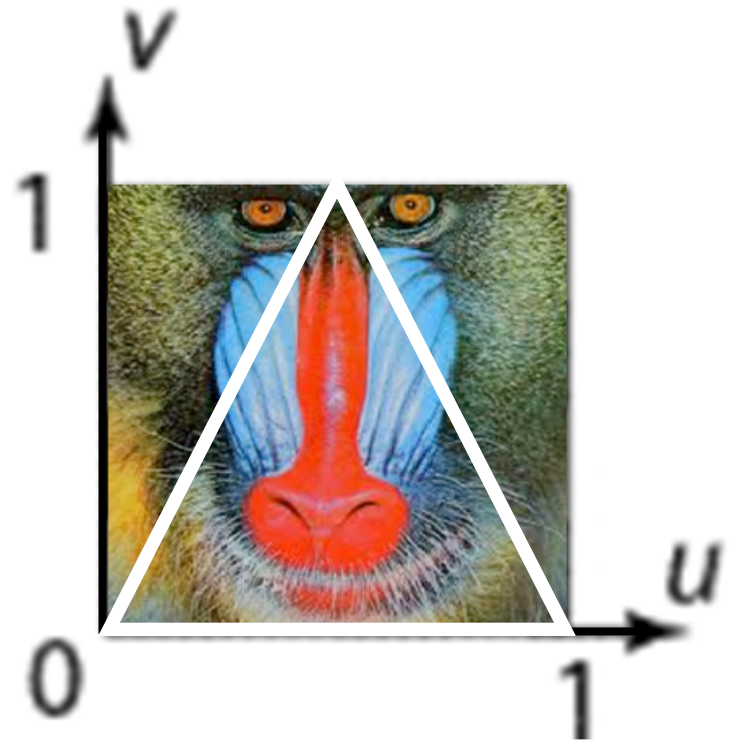
// Activate and bind texture to the multi-texture target 0
glActiveTexture(GL_TEXTURE0);
glBindTexture(GL_TEXTURE_2D, mTexId);

glBegin(GL_TRIANGLES)
    glTexCoord2f(0, 0);
    glVertex3f(-10,0,0);

    glTexCoord2f(1, 0);
    glVertex3f(10,0,0);

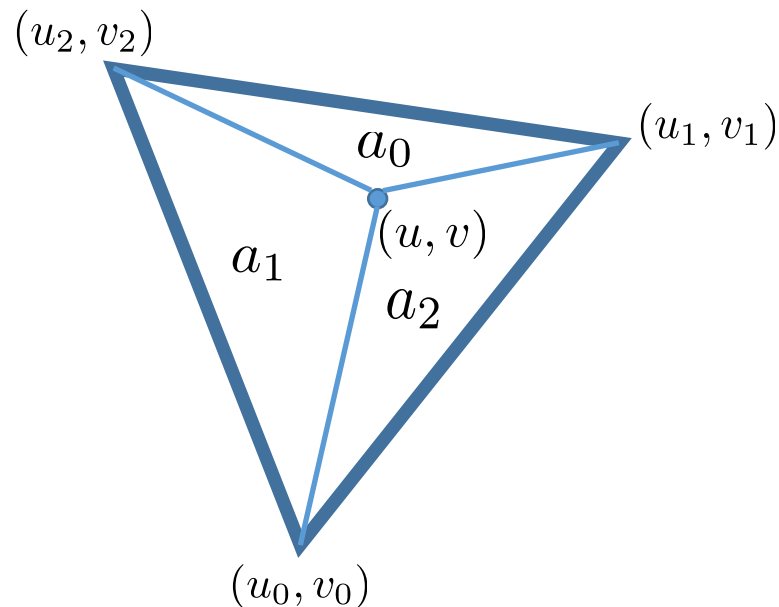
    glTexCoord2f(0.5, 1);
    glVertex3f(5,10,0);
glEnd()

// Destroy texture once you are completely done with it.
glDeleteTextures(1, &mTexId);
```



Texture Coordinates

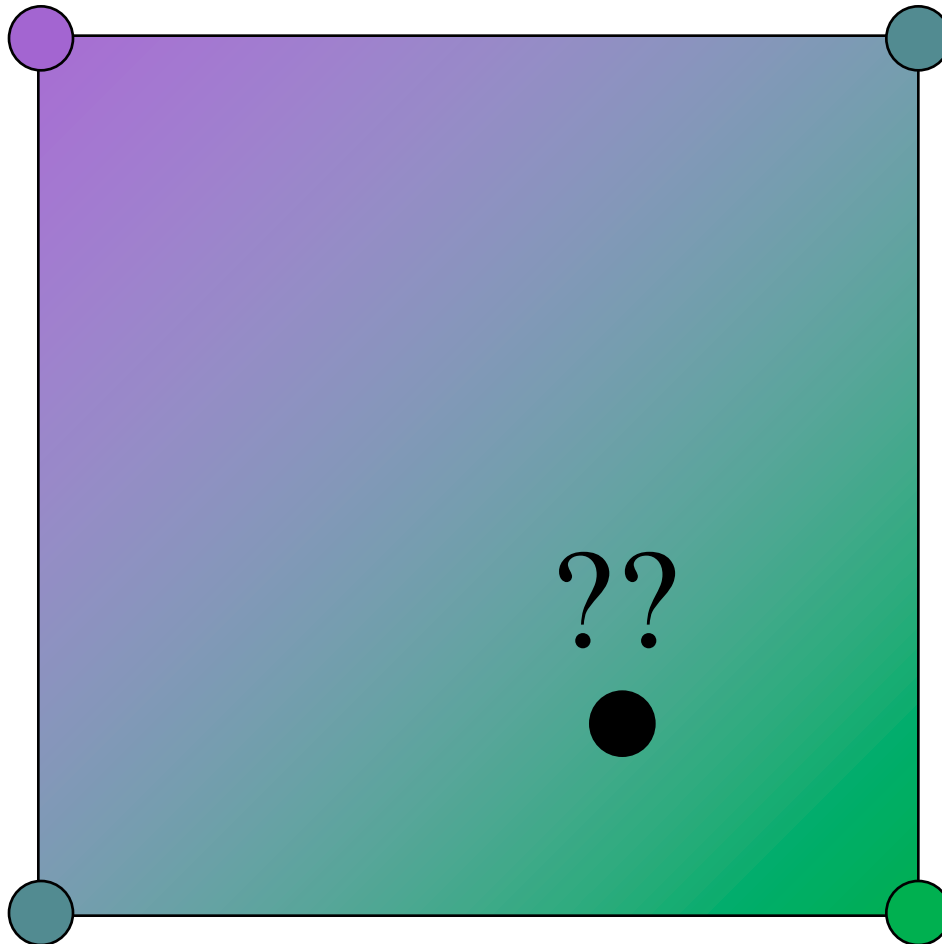
- Then, for each pixel inside a triangle, calculate the pixel's (u,v) texture coordinates using barycentric interpolation of the triangle vertices' texture coordinates



$$[u, v] = a_0[u_0, v_0] + a_1[u_1, v_1] + a_2[u_2, v_2]$$

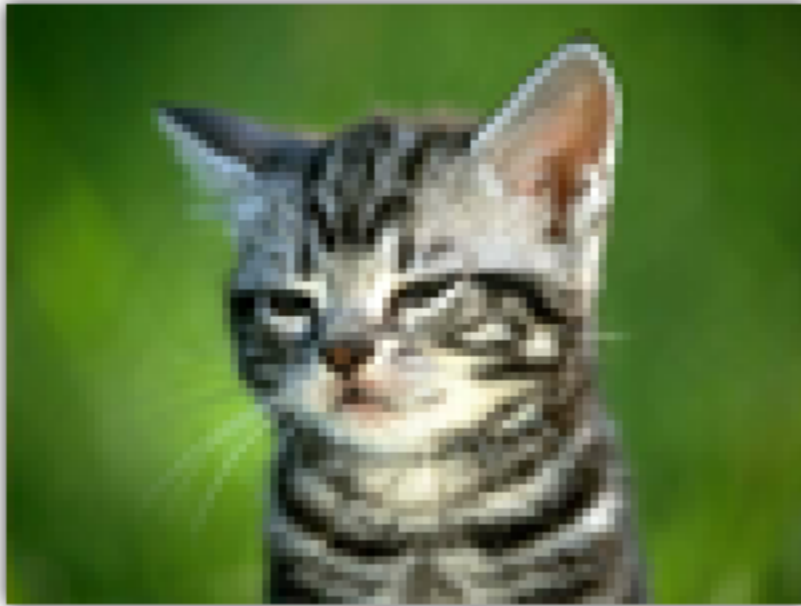
Pixel Color

- Given the pixel's (u,v) texture coordinates, use interpolation in the texture map to find the pixel's color



**Between four
pixels!**

Pixel Color: Nearest Neighbor



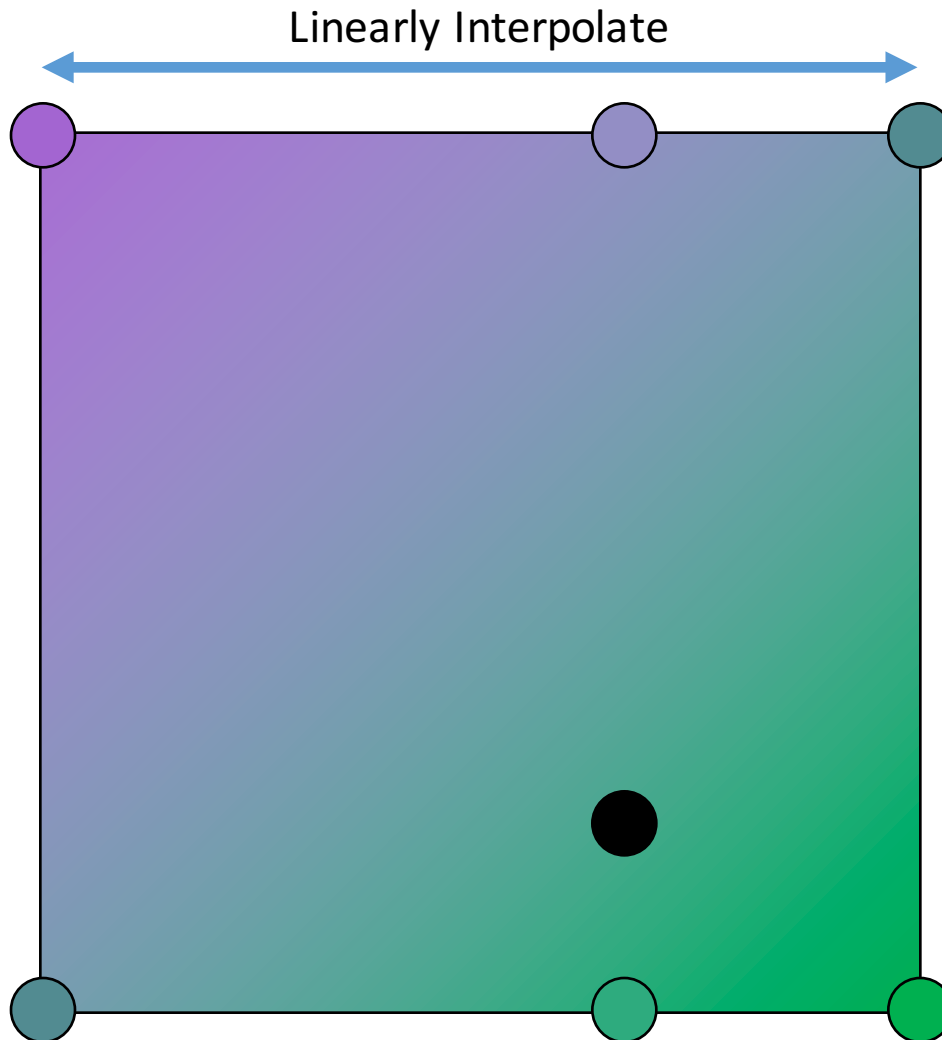
GL_NEAREST



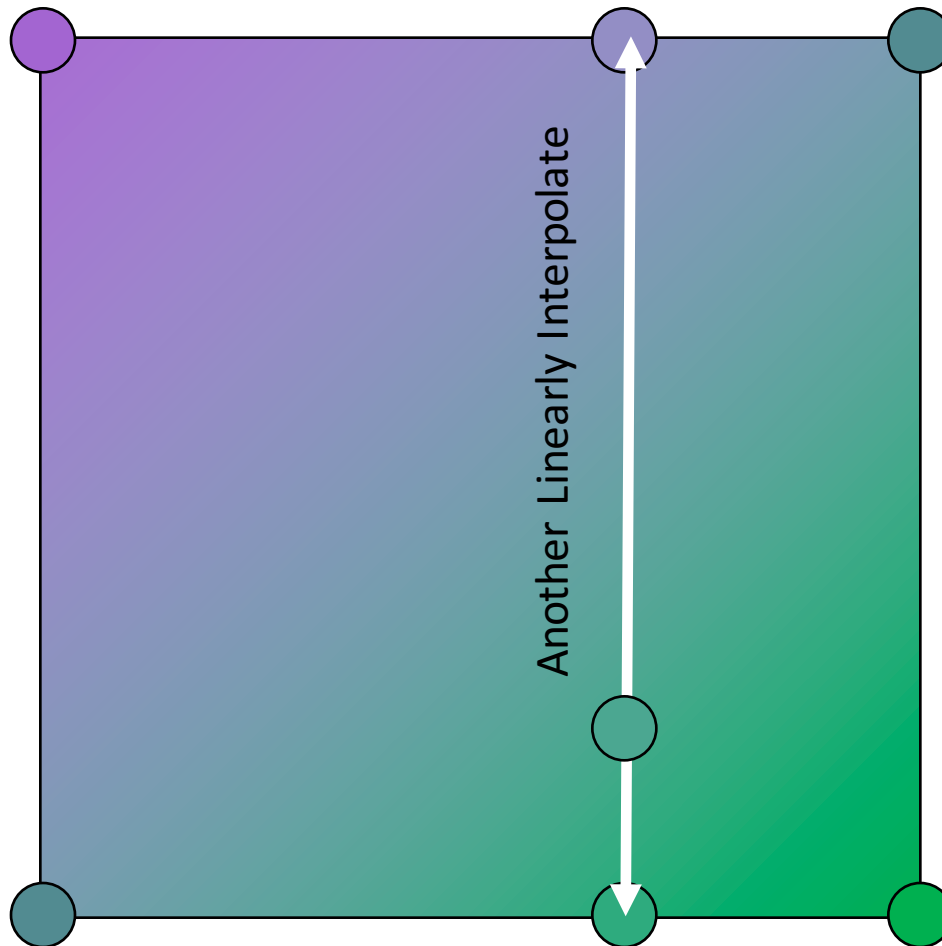
GL_LINEAR



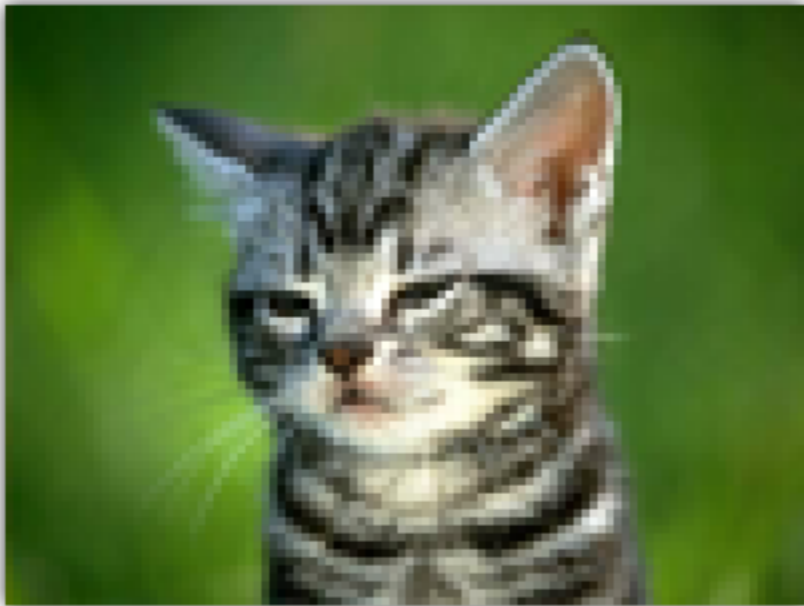
Pixel Color: Bilinear Interpolation



Pixel Color: Bilinear Interpolation



Nearest Neighbor Vs Bilinear

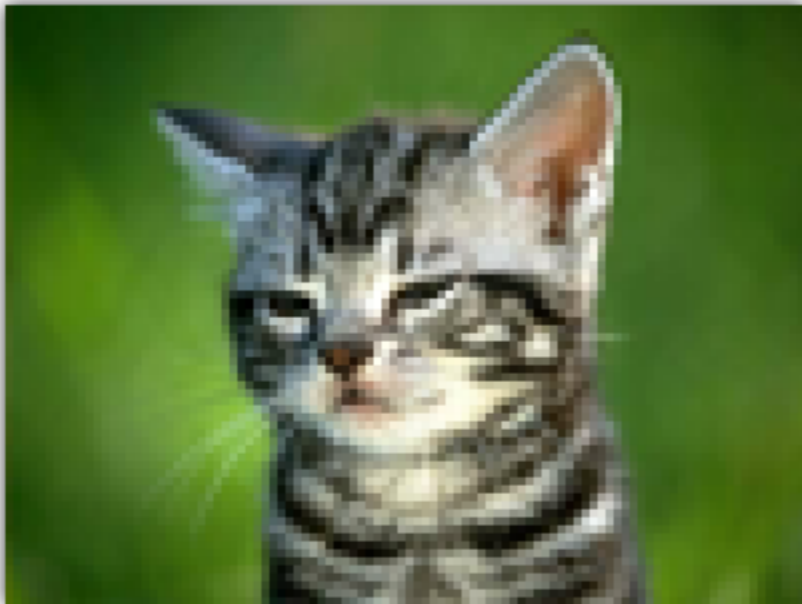


GL_NEAREST



GL_LINEAR

Nearest Neighbor Vs Bilinear



GL_NEAREST

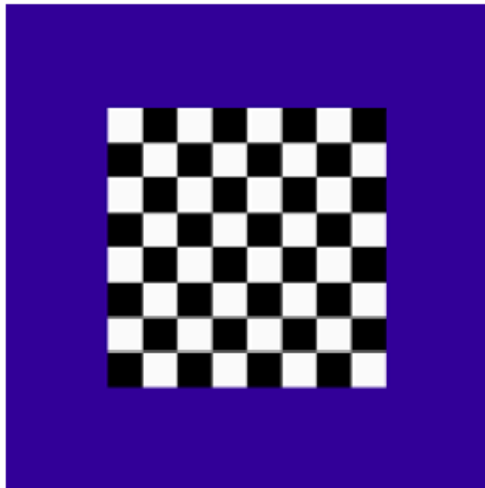


GL_LINEAR

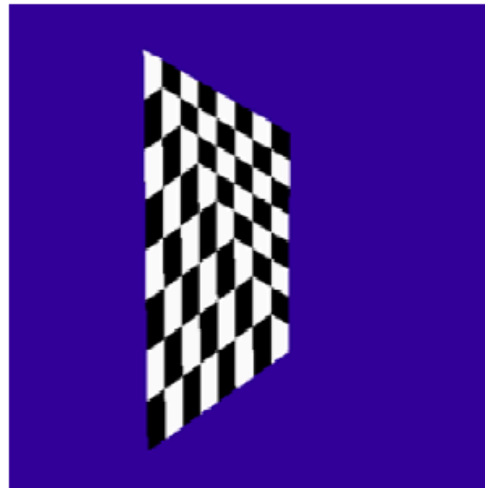
More on this when we discuss "Sampling"

Screen Space vs. World Space

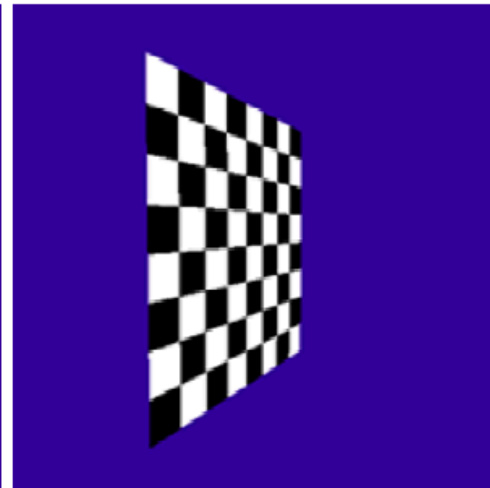
- Triangles change shape nonlinearly via perspective transformation, leading to different barycentric weights before and after the perspective transformation
- Interpolating in screen space results in texture distortion
- Interpolating in world space requires projecting all pixel locations backwards from screen space to world space, which is expensive



texture source

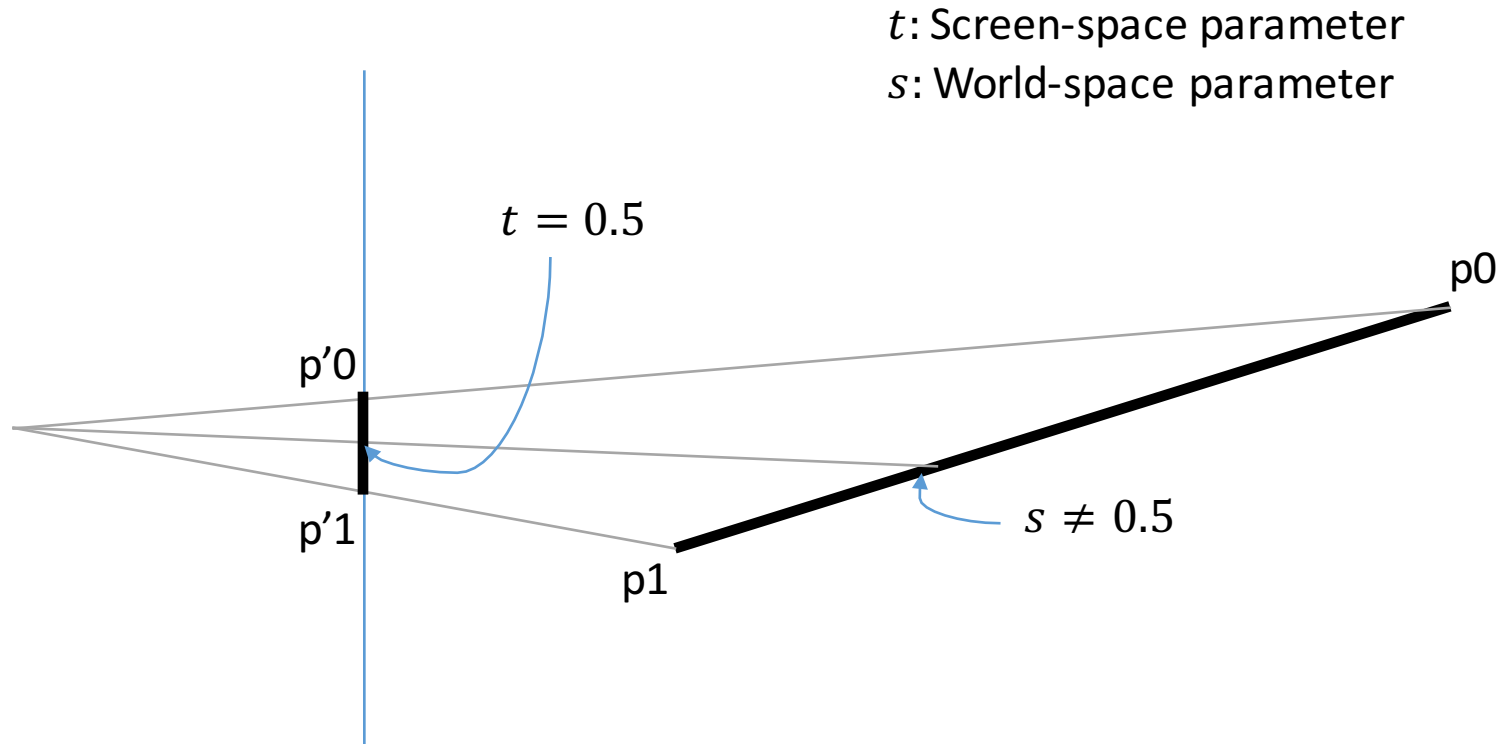


what we get

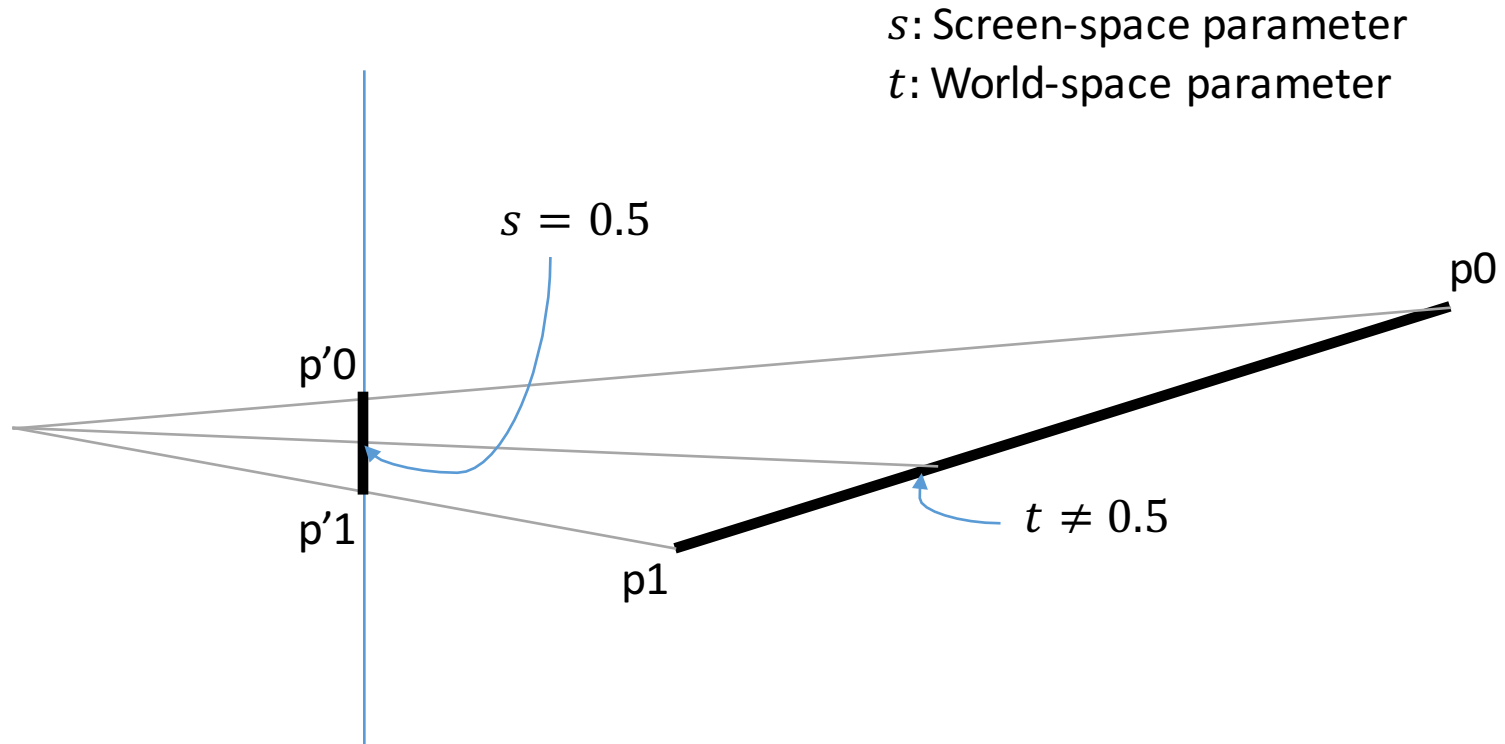


what we want

Texture Distortion



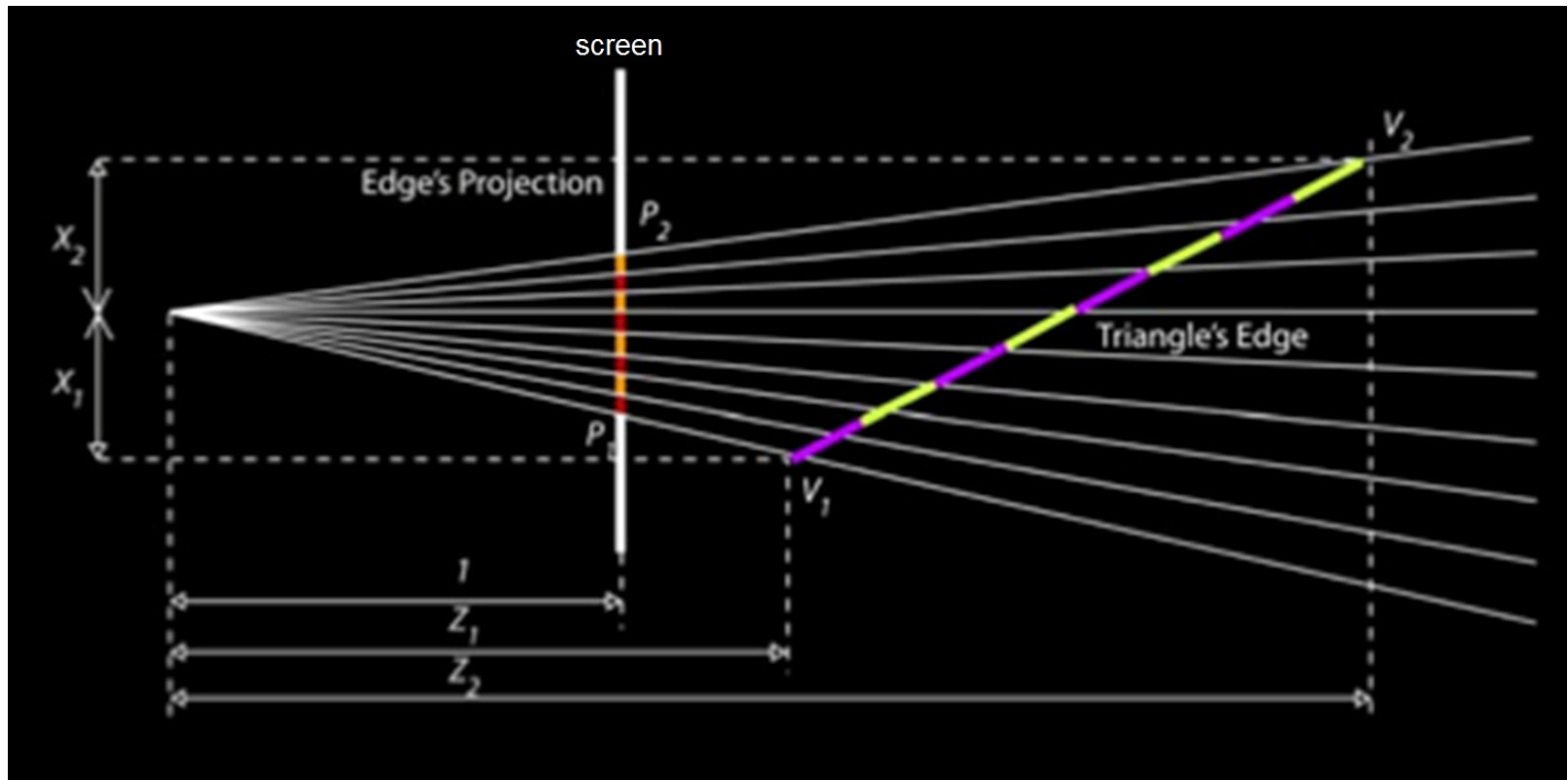
Texture Distortion



**Screen-space and World-space parameters
don't match !**

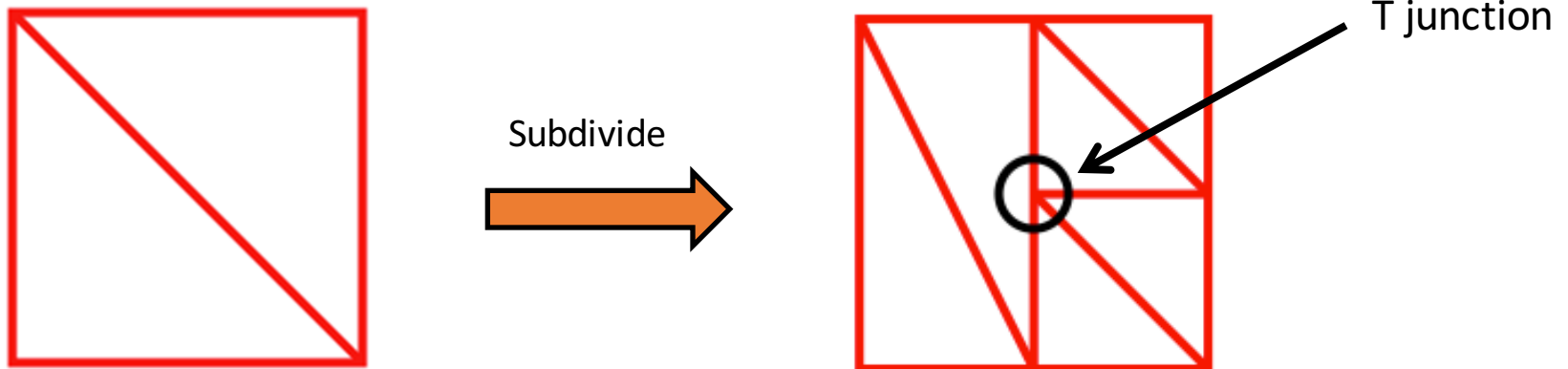
Texture Distortion

- Uniform increments along the edge in world space do not correspond to uniform increments along the edge in screen space
- Barycentric interpolation (which is linear) does not account for this nonlinearity

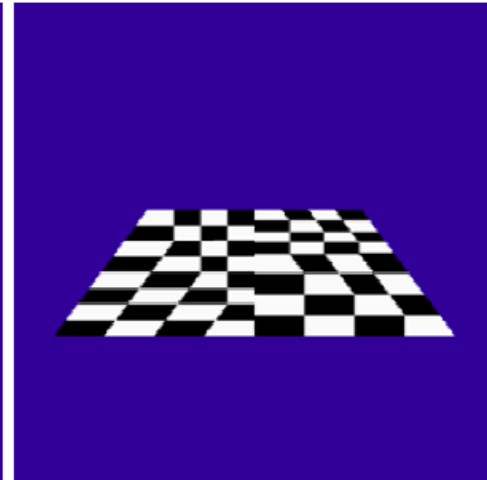
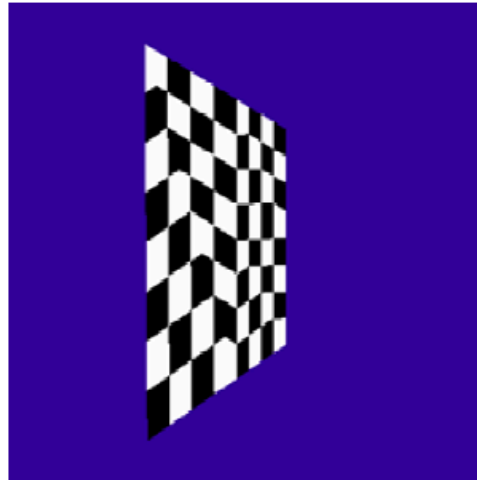
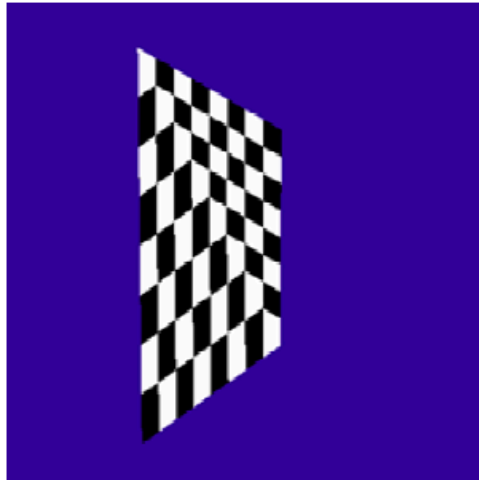
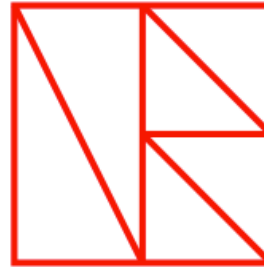


Mesh Refinement

- Refinement of the triangle mesh improves the result
- A nonlinear function can be approximated as a piecewise linear function if the intervals are small enough
- However some errors are still obvious, especially at T-junctions where levels-of-refinement change



Mesh Refinement



Does not work !

Perspective Correct Interpolation

- Find the relationship between the barycentric weights in screen space and those in world space
- Use this relationship to compute the world space barycentric weights from the screen space barycentric weights

Perspective Correct Interpolation

Two points in world space

$$p_1^w = \begin{bmatrix} x_1 \\ z_1 \end{bmatrix} \quad p_2^w = \begin{bmatrix} x_2 \\ z_2 \end{bmatrix}$$

Perspective Correct Interpolation

Two points in world space

$$p_1^w = \begin{bmatrix} x_1 \\ z_1 \end{bmatrix} \quad p_2^w = \begin{bmatrix} x_2 \\ z_2 \end{bmatrix}$$

Interpolation in world space

$$p^w(t) = (1 - t) \begin{bmatrix} x_1 \\ z_1 \end{bmatrix} + t \begin{bmatrix} x_2 \\ z_2 \end{bmatrix}$$

Perspective Correct Interpolation

Two points in world space

$$p_1^w = \begin{bmatrix} x_1 \\ z_1 \end{bmatrix} \quad p_2^w = \begin{bmatrix} x_2 \\ z_2 \end{bmatrix}$$

Interpolation in world space

$$p^w(t) = (1 - t) \begin{bmatrix} x_1 \\ z_1 \end{bmatrix} + t \begin{bmatrix} x_2 \\ z_2 \end{bmatrix}$$

Project the interpolated point on the screen

$$\text{Proj}(p_x^w(t)) = d \frac{(1 - t)x_1 + tx_2}{(1 - t)z_1 + tz_2}$$

Perspective Correct Interpolation

Two points in world space

$$p_1^w = \begin{bmatrix} x_1 \\ z_1 \end{bmatrix} \quad p_2^w = \begin{bmatrix} x_2 \\ z_2 \end{bmatrix}$$

Interpolation in world space

$$p^w(t) = (1 - t) \begin{bmatrix} x_1 \\ z_1 \end{bmatrix} + t \begin{bmatrix} x_2 \\ z_2 \end{bmatrix}$$

Project the interpolated point on the screen

$$\text{Proj}(p_x^w(t)) = d \frac{(1 - t)x_1 + tx_2}{(1 - t)z_1 + tz_2}$$

Recall Projection!

Perspective Correct Interpolation

Interpolation of the same two points in screen space (after projection)

$$P_x^s(s) = (1 - s) \frac{dx_1}{z_1} + s \frac{dx_2}{z_2}$$

Perspective Correct Interpolation

Interpolation of the same two points in screen space (after projection)

$$P_x^s(s) = (1 - s) \frac{dx_1}{z_1} + s \frac{dx_2}{z_2}$$

Screen space point and world-space point after projection must match

$$d \frac{(1 - t)x_1 + tx_2}{(1 - t)z_1 + tz_2} = (1 - s) \frac{dx_1}{z_1} + s \frac{dx_2}{z_2}$$

Perspective Correct Interpolation

Interpolation of the same two points in screen space (after projection)

$$P_x^s(s) = (1 - s) \frac{dx_1}{z_1} + s \frac{dx_2}{z_2}$$

Screen space point and world-space point after projection must match

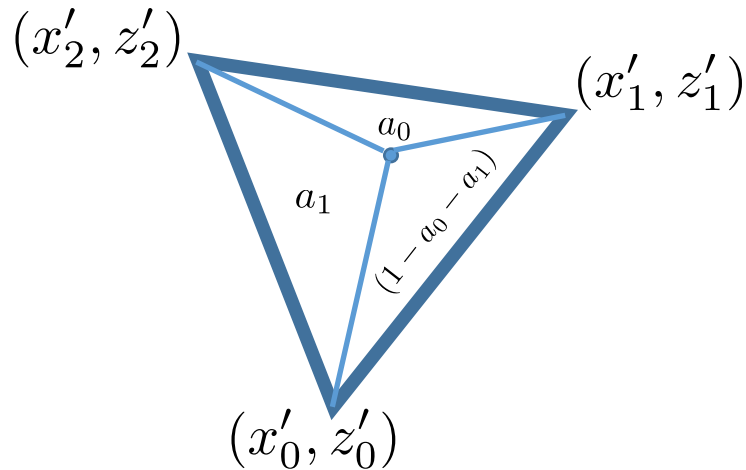
$$d \frac{(1 - t)x_1 + tx_2}{(1 - t)z_1 + tz_2} = (1 - s) \frac{dx_1}{z_1} + s \frac{dx_2}{z_2}$$

After algebra

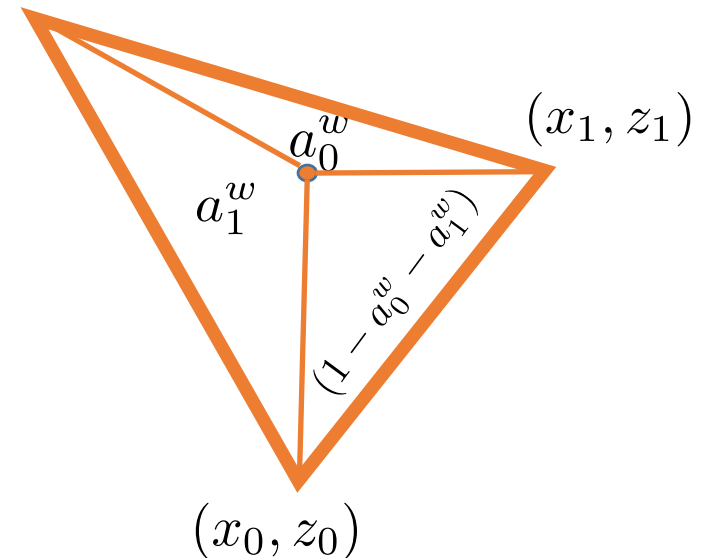
$$t = \frac{sz_1}{z_2 + s(z_1 - z_2)}$$

Perspective Correct Interpolation

Screen Space Triangle



(x_2, z_2) World Space Triangle



$$a_0^w = \frac{z_1 z_2 a_0}{a_0 z_1 z_2 + a_1 z_0 z_2 + (1 - a_0 - a_1) z_0 z_1}$$

$$a_1^w = \frac{z_0 z_2 a_1}{a_1 z_0 z_2 + a_0 z_1 z_2 + (1 - a_0 - a_1) z_0 z_1}$$

Perspective Correct Interpolation

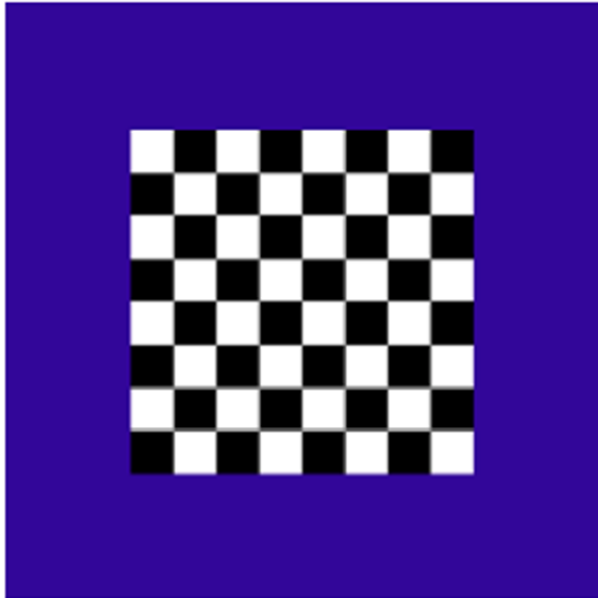
Finally!

$$[u, v] = a_0^w [u_0, v_0] + a_1^w [u_1, v_1] + a_2^w [u_2, v_2]$$

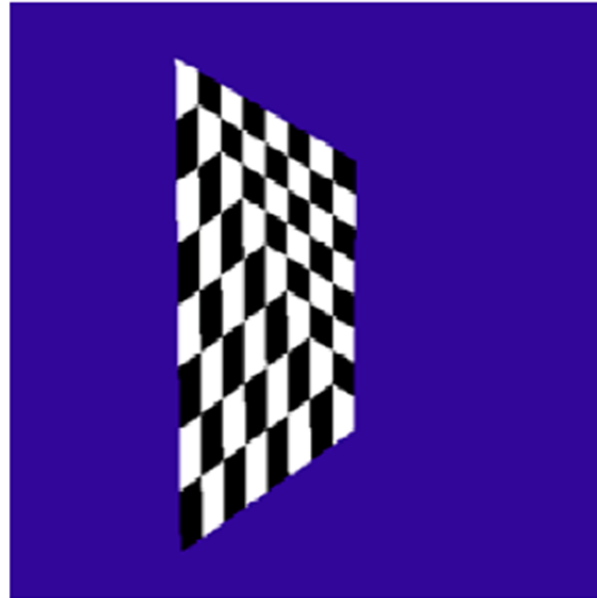
Perspective Correct Interpolation

Finally!

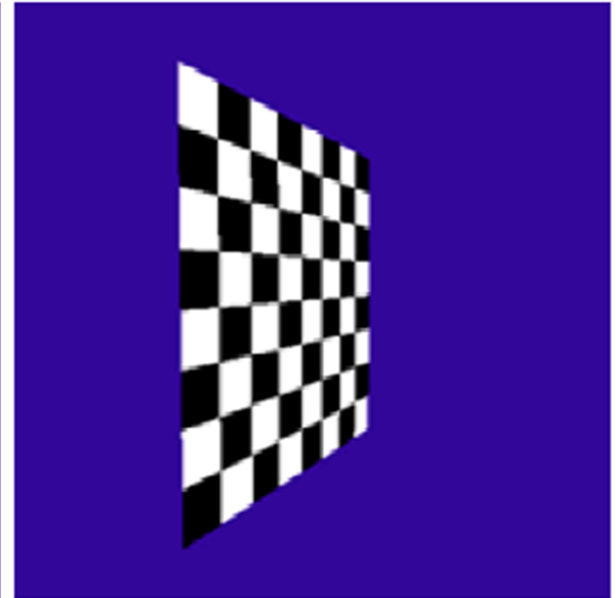
$$[u, v] = a_0^w [u_0, v_0] + a_1^w [u_1, v_1] + a_2^w [u_2, v_2]$$



texture source

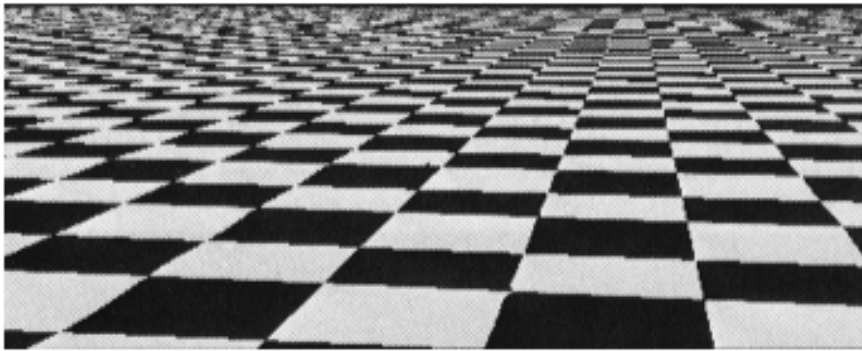


results without
perspective correct interpolation

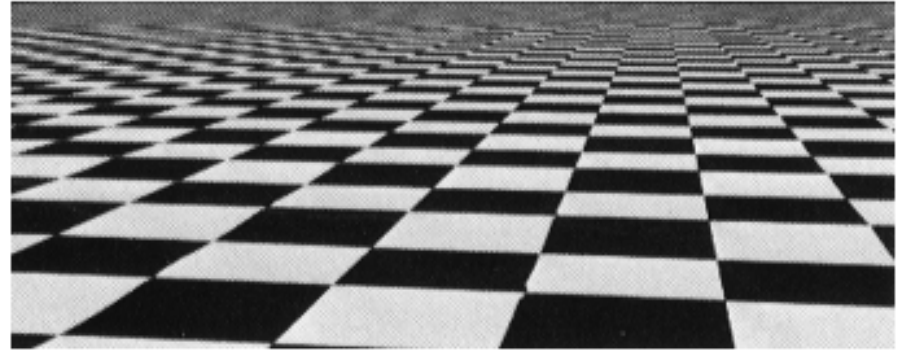


results with
perspective correct interpolation

Aliasing



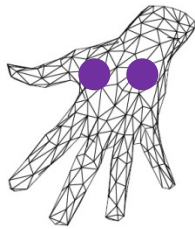
What we get



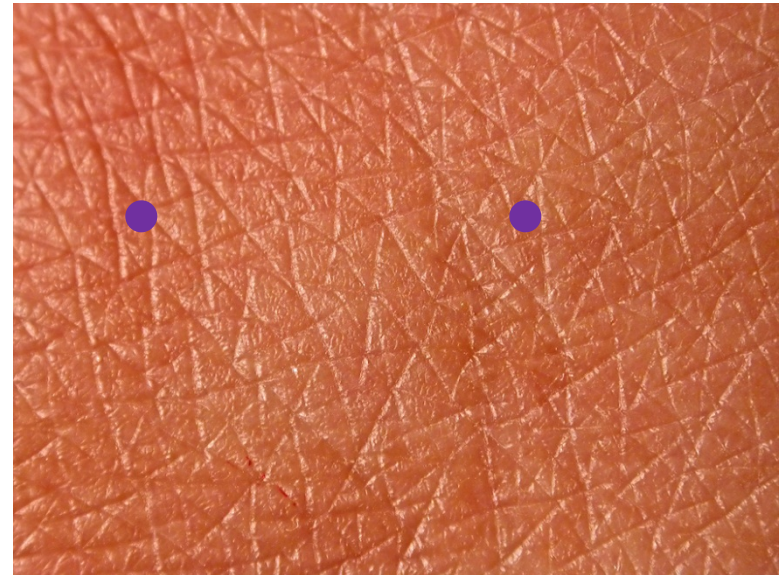
What we want

Aliasing

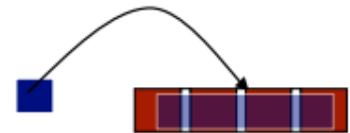
Small image



Large texture

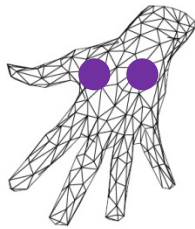


Source pixels covers many destination pixels

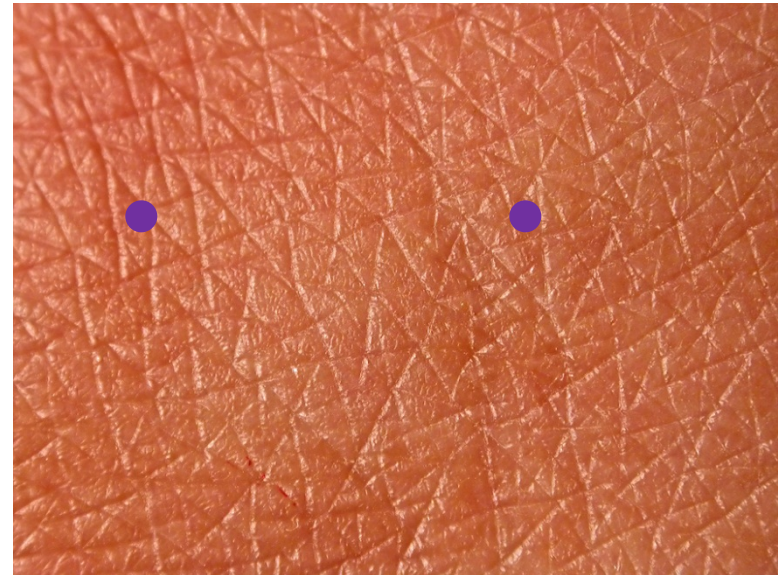


Aliasing

Small image

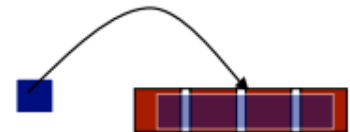


Large texture



**More when we
discuss sampling**

Source pixels covers many destination pixels



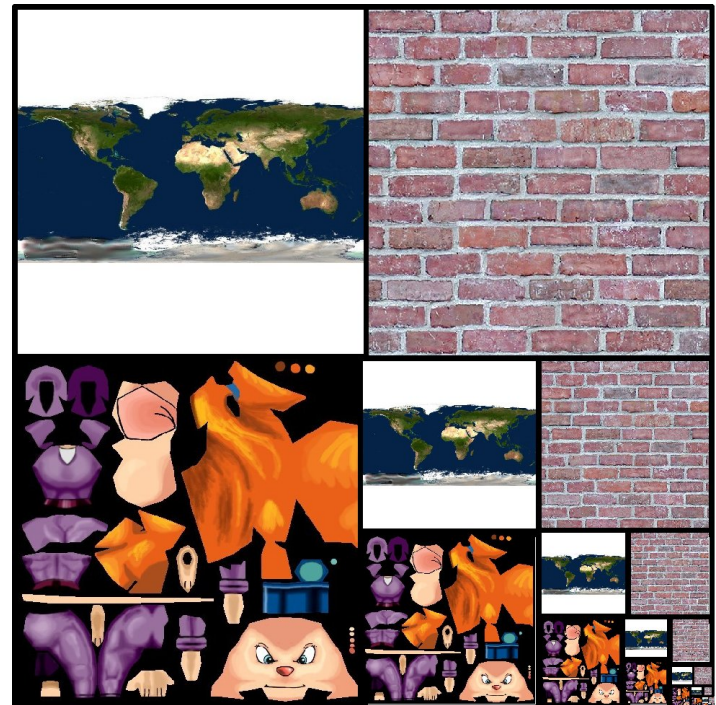
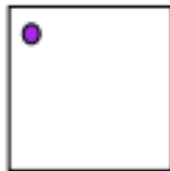
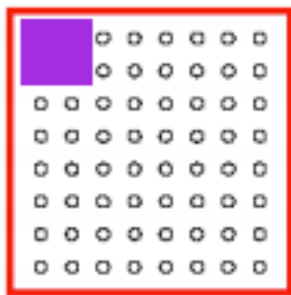
MIP Maps

- Multum in Parvo: Much in little, many in small places
- Precomputes the texture maps at multiple resolutions, using averaging as a low pass filter
- When texture mapping, choose the image size that approximately gives a 1 to 1 pixel to texel correspondence
- The averaging “bakes-in” all the nearby pixels that otherwise would not be sampled correctly



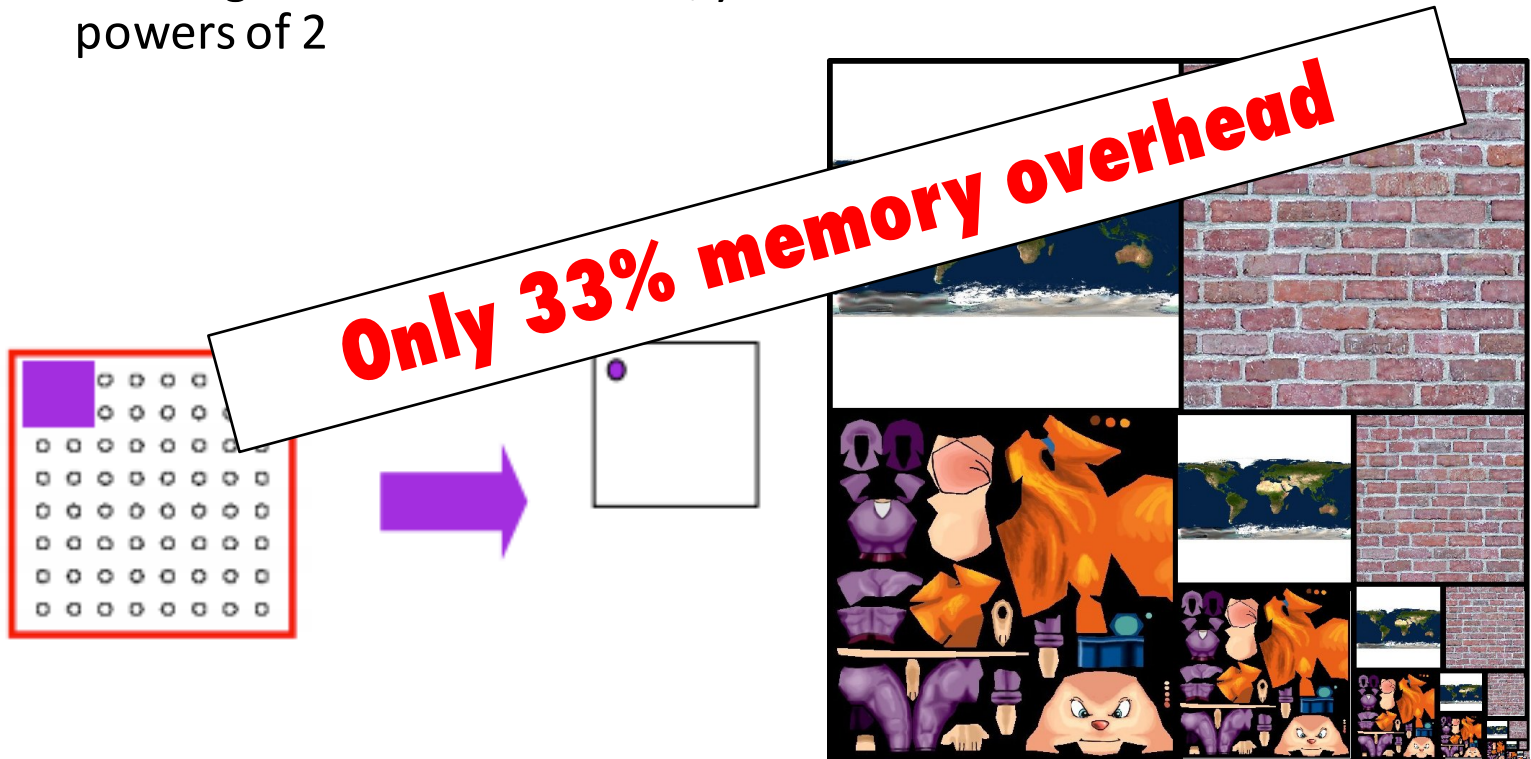
MIP Maps

- 4 neighboring pixels of the higher level are averaged to form a single pixel in the lower level
- Starting at a base resolution, you can store EVERY coarser resolution in powers of 2

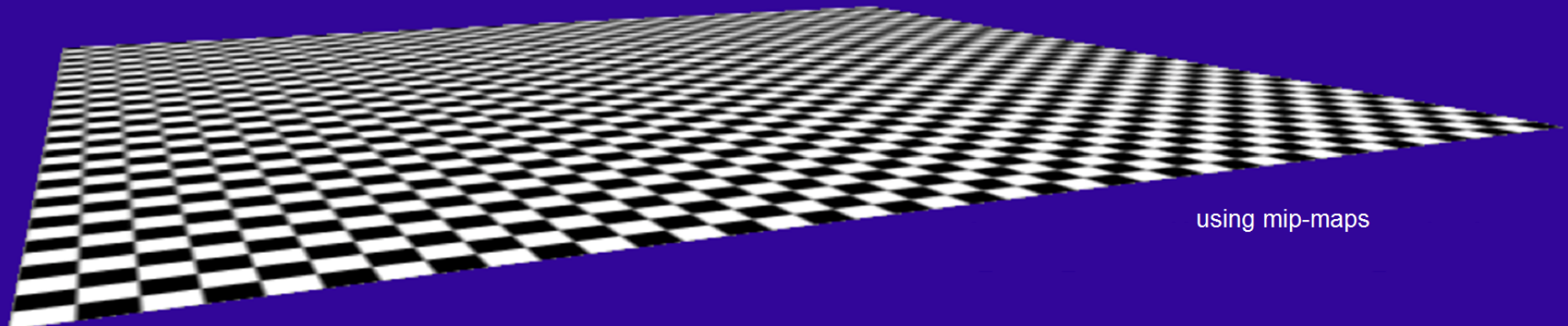
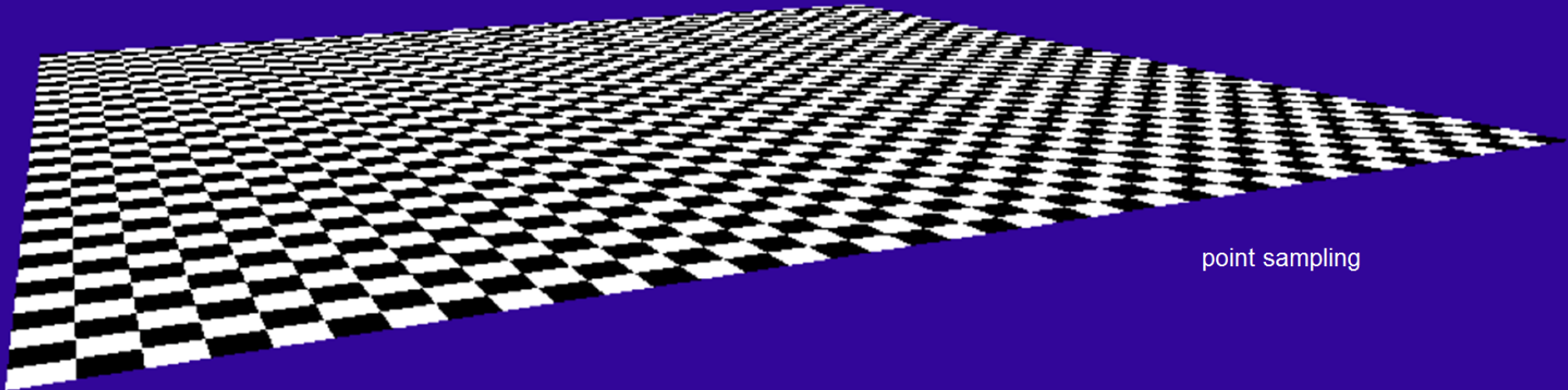


MIP Maps

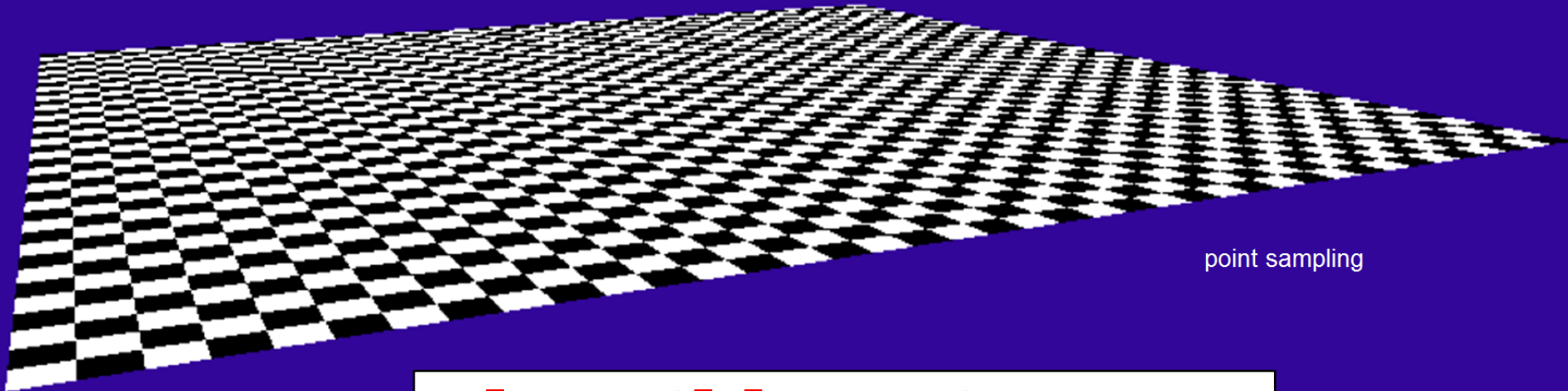
- 4 neighboring pixels of the higher level are averaged to form a single pixel in the lower level
- Starting at a base resolution, you can store EVERY coarser resolution in powers of 2



MIP Maps



MIP Maps



point sampling

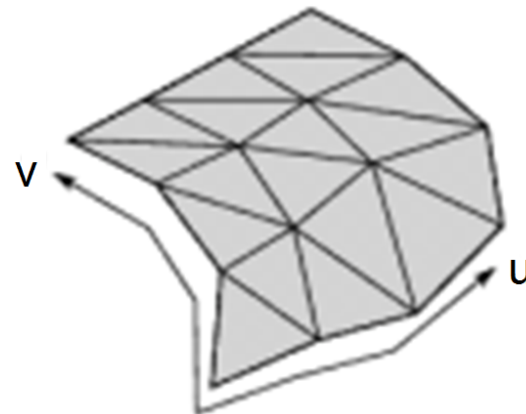
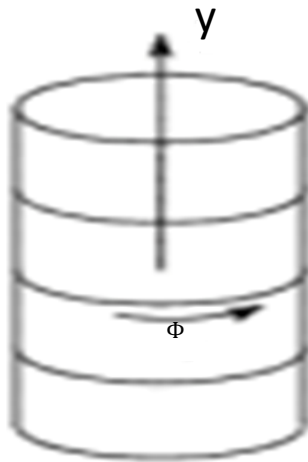
gluBuild2DMipmaps



using mip-maps

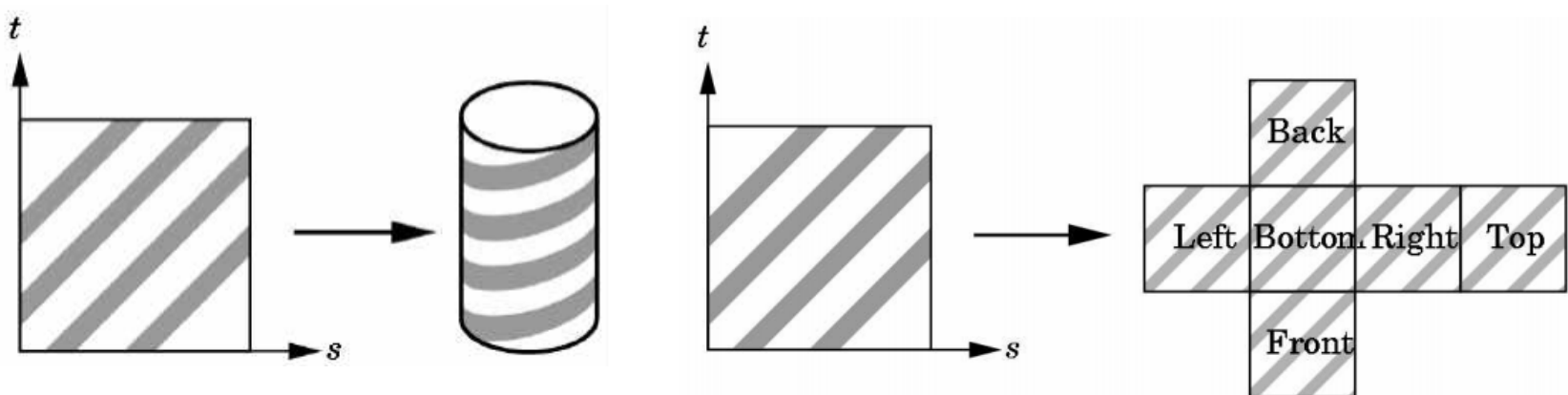
Assigning Texture Coordinates

- For certain surfaces, the (u, v) texture coordinates can be generated procedurally
- Example: Cylinder
 - map the u coordinate from $[0, 1]$ to $[0, 2\pi]$ for Φ
 - map the v coordinate from $[0, 1]$ to $[0, h]$ for y
 - This wraps the image around the cylinder
- For more complex surfaces, (u, v) must be defined per vertex manually or by using proxy objects



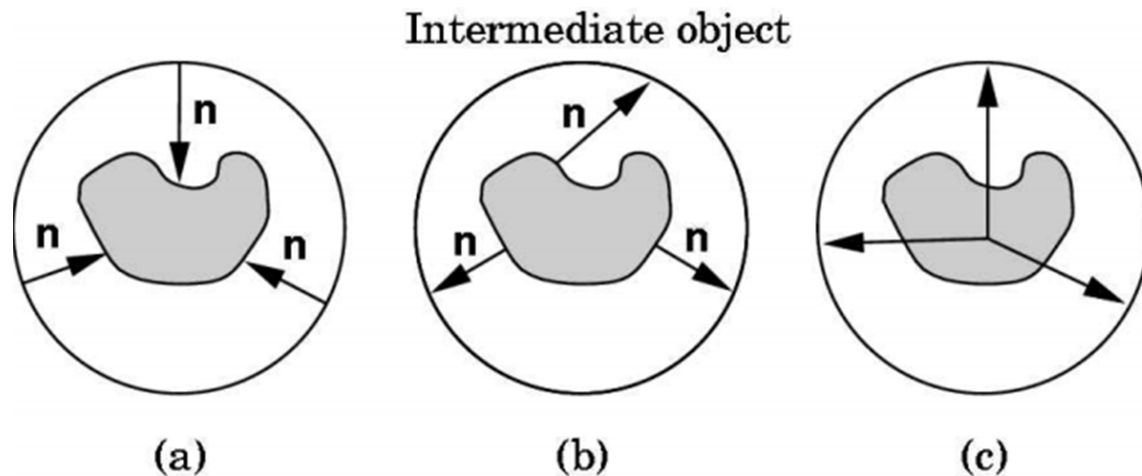
Proxy Objects – Step 1

- Assign texture coordinates to intermediate/proxy objects:
 - Example: Cylinder
 - wrap texture around the outside of the cylinder
 - not the top or bottom, in order to avoid distorting the texture
 - Example: Cube
 - unwrap the cube and map texture over the unwrapped cube
 - the texture is seamless across some of the edges, but not necessarily others

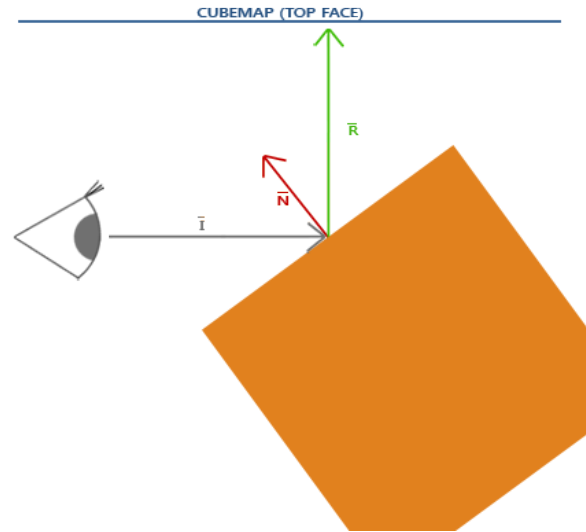


Proxy Objects – Step 2

- Map texture coordinates from the intermediate/proxy object to the final object
- Three ways of mapping are typically used
 - Use the intermediate/proxy object's surface normal
 - Use the target object's surface normal
 - Use rays emanating from center of target object



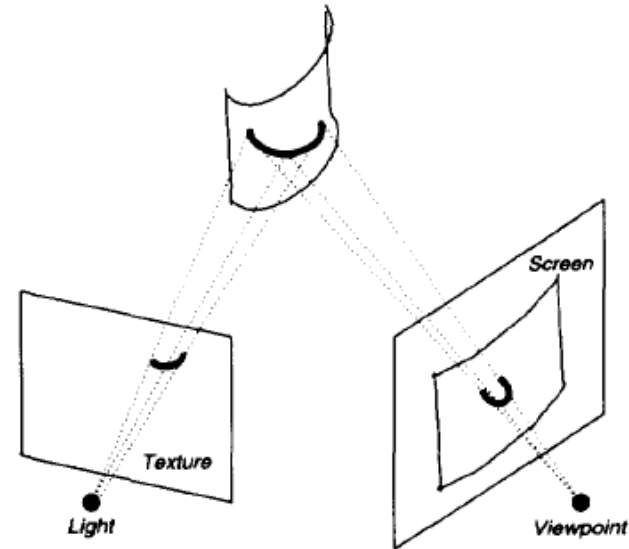
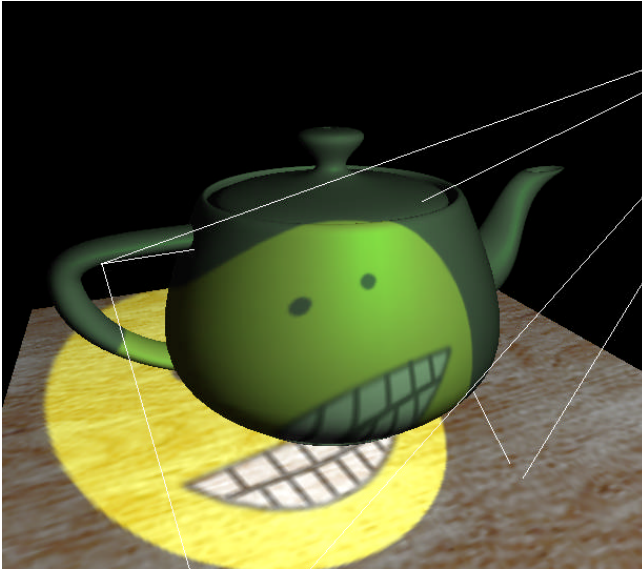
Cube Mapping



```
glTexGenfv(GL_S, GL_TEXTURE_GEN_MODE, GL_REFLECTION_MAP_EXT);  
glTexGenfv(GL_T, GL_TEXTURE_GEN_MODE, GL_REFLECTION_MAP_EXT);  
glTexGenfv(GL_R, GL_TEXTURE_GEN_MODE, GL_REFLECTION_MAP_EXT);  
glEnable(GL_TEXTURE_GEN_S);  
glEnable(GL_TEXTURE_GEN_T);  
glEnable(GL_TEXTURE_GEN_R);
```

<http://learnopengl.com/#!Advanced-OpenGL/Cubemaps>

Projective Texturing



- Treat light Source as a
- Render the scene normally from the actual camera

http://www.nvidia.com/object/Projective_Texture_Mapping.html

Segal et. al. SIGGRAPH'92

Projective Texturing

- Assign Texture Coordinates (s,t,r) to position (x,y,z)

```
glTexGeni(GL_S, GL_TEXTURE_GEN_MODE, GL_OBJECT_LINEAR);
glTexGeni(GL_T, GL_TEXTURE_GEN_MODE, GL_OBJECT_LINEAR);
glTexGeni(GL_R, GL_TEXTURE_GEN_MODE, GL_OBJECT_LINEAR);

float [] planeS = { 1.0f, 0.0f, 0.0f, 0.0f };
glTexGenfv(GL_S, GL_OBJECT_PLANE, planeS);

float [] planeT = { 0.0f, 1.0f, 0.0f, 0.0f };
glTexGenfv(GL_T, GL_OBJECT_PLANE, planeT);

float [] planeR = { 0.0f, 0.0f, 1.0f, 0.0f };
glTexGenfv(GL_R, GL_OBJECT_PLANE, planeR);

glEnable(GL_TEXTURE_GEN_S);
glEnable(GL_TEXTURE_GEN_T);
glEnable(GL_TEXTURE_GEN_R);
```

$$\begin{bmatrix} s \\ t \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ObjectSpace}$$

Projective Texturing

- Assign Texture Coordinates (s,t,r) to position (x,y,z)

```
glTexGeni(GL_S, GL_TEXTURE_GEN_MODE, GL_OBJECT_LINEAR);
glTexGeni(GL_T, GL_TEXTURE_GEN_MODE, GL_OBJECT_LINEAR);
glTexGeni(GL_R, GL_TEXTURE_GEN_MODE, GL_OBJECT_LINEAR);

float [] planeS = { 1.0f, 0.0f, 0.0f, 0.0f };
glTexGenfv(GL_S, GL_OBJECT_PLANE, planeS);

float [] planeT = { 0.0f, 1.0f, 0.0f, 0.0f };
glTexGenfv(GL_T, GL_OBJECT_PLANE, planeT);

float [] planeR = { 0.0f, 0.0f, 1.0f, 0.0f };
glTexGenfv(GL_R, GL_OBJECT_PLANE, planeR);

glEnable(GL_TEXTURE_GEN_S);
glEnable(GL_TEXTURE_GEN_T);
glEnable(GL_TEXTURE_GEN_R);
```

$$\begin{bmatrix} s \\ t \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ ObjectSpace}$$

Projective Texturing

- Assign Texture Coordinates (s,t,r) to position (x,y,z)

```
glTexGeni(GL_S, GL_TEXTURE_GEN_MODE, GL_OBJECT_LINEAR);
glTexGeni(GL_T, GL_TEXTURE_GEN_MODE, GL_OBJECT_LINEAR);
glTexGeni(GL_R, GL_TEXTURE_GEN_MODE, GL_OBJECT_LINEAR);

float [] planeS = { 1.0f, 0.0f, 0.0f, 0.0f };
glTexGenfv(GL_S, GL_OBJECT_PLANE, planeS);

float [] planeT = { 0.0f, 1.0f, 0.0f, 0.0f };
glTexGenfv(GL_T, GL_OBJECT_PLANE, planeT);

float [] planeR = { 0.0f, 0.0f, 1.0f, 0.0f };
glTexGenfv(GL_R, GL_OBJECT_PLANE, planeR);

glEnable(GL_TEXTURE_GEN_S);
glEnable(GL_TEXTURE_GEN_T);
glEnable(GL_TEXTURE_GEN_R);
```

$$\begin{bmatrix} s \\ t \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ ObjectSpace}$$

Projective Texturing

- Assign Texture Coordinates (s,t,r) to position (x,y,z)

```
glTexGeni(GL_S, GL_TEXTURE_GEN_MODE, GL_OBJECT_LINEAR);  
glTexGeni(GL_T, GL_TEXTURE_GEN_MODE, GL_OBJECT_LINEAR);  
glTexGeni(GL_R, GL_TEXTURE_GEN_MODE, GL_OBJECT_LINEAR);  
  
float [] planeS = { 1.0f, 0.0f, 0.0f, 0.0f };  
glTexGenfv(GL_S, GL_OBJECT_PLANE, planeS);
```

**So much work just to say $(s,t,r) = (x,y,z)$
Much easily done in newer OpenGL**

```
glEnable(GL_TEXTURE_GEN_S);  
glEnable(GL_TEXTURE_GEN_T);  
glEnable(GL_TEXTURE_GEN_R);
```

$$\begin{bmatrix} s \\ t \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ ObjectSpace}$$

Projective Texturing

- Use a similar View (from the light's point of view) and Projection matrix to transform texture coordinates to NDC (-1 , 1)

$$\begin{bmatrix} s' \\ t' \\ r' \\ q' \end{bmatrix}_{NDC} = P \cdot V \cdot M \begin{bmatrix} s \\ t \\ r \\ 1 \end{bmatrix}$$

Projective Texturing

- Map NDC (-1 , 1) to Texture Coordinate space (0-1)
 - Scale and add Bias

$$\begin{bmatrix} s'' \\ t'' \\ r'' \\ q'' \end{bmatrix}_{\text{TextureSpace}} = \begin{bmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s' \\ t' \\ r' \\ q' \end{bmatrix}_{\text{NDC}}$$

Projective Texturing

- Map NDC (-1 , 1) to Texture Coordinate space (0-1)
 - Scale and add Bias

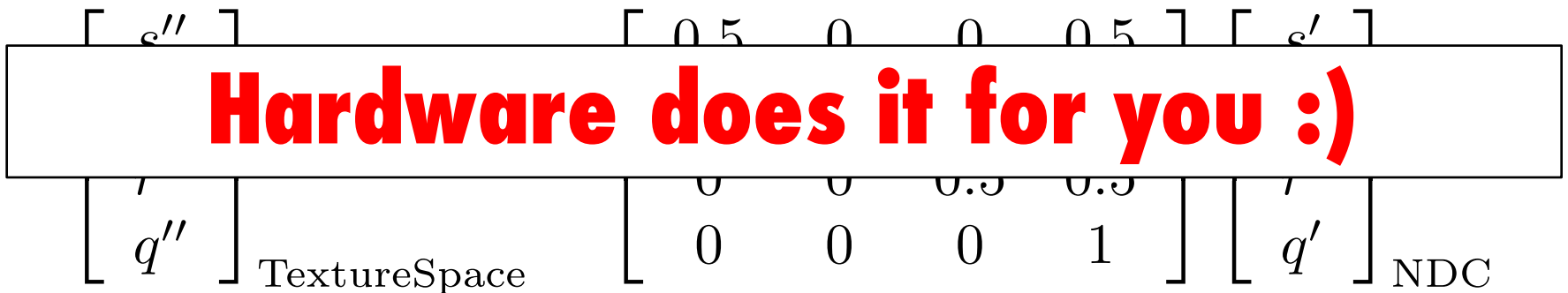
$$\begin{bmatrix} s'' \\ t'' \\ r'' \\ q'' \end{bmatrix}_{\text{TextureSpace}} = \begin{bmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s' \\ t' \\ r' \\ q' \end{bmatrix}_{\text{NDC}}$$

Final texture coordinates after perspective-correct interpolation of (s'', t'', r'', q'')

$$\left(\frac{s''}{q''}, \frac{t''}{q''}, \frac{r''}{q''} \right)$$

Projective Texturing

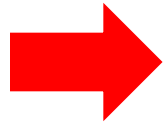
- Map NDC (-1 , 1) to Texture Coordinate space (0-1)
 - Scale and add Bias



Final texture coordinates after perspective-correct interpolation of (s'', t'', r'', q'')

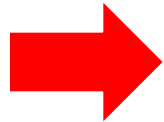
$$\left(\frac{s''}{q''}, \frac{t''}{q''}, \frac{r''}{q''} \right)$$

How To Set Texture Matrices ?



```
glMatrixMode(GL_TEXTURE);  
glLoadIdentity();  
glTranslatef(0.5,0.5,0.5);  
glScale3f(0.5,0.5,0.5);  
gluPerspective(...);  
gluLookAt(...);  
glLoadMatrixf(modelMatrix);  
.  
.  
glMatrixMode(GL_MODELVIEW);
```

How To Set Texture Matrices ?

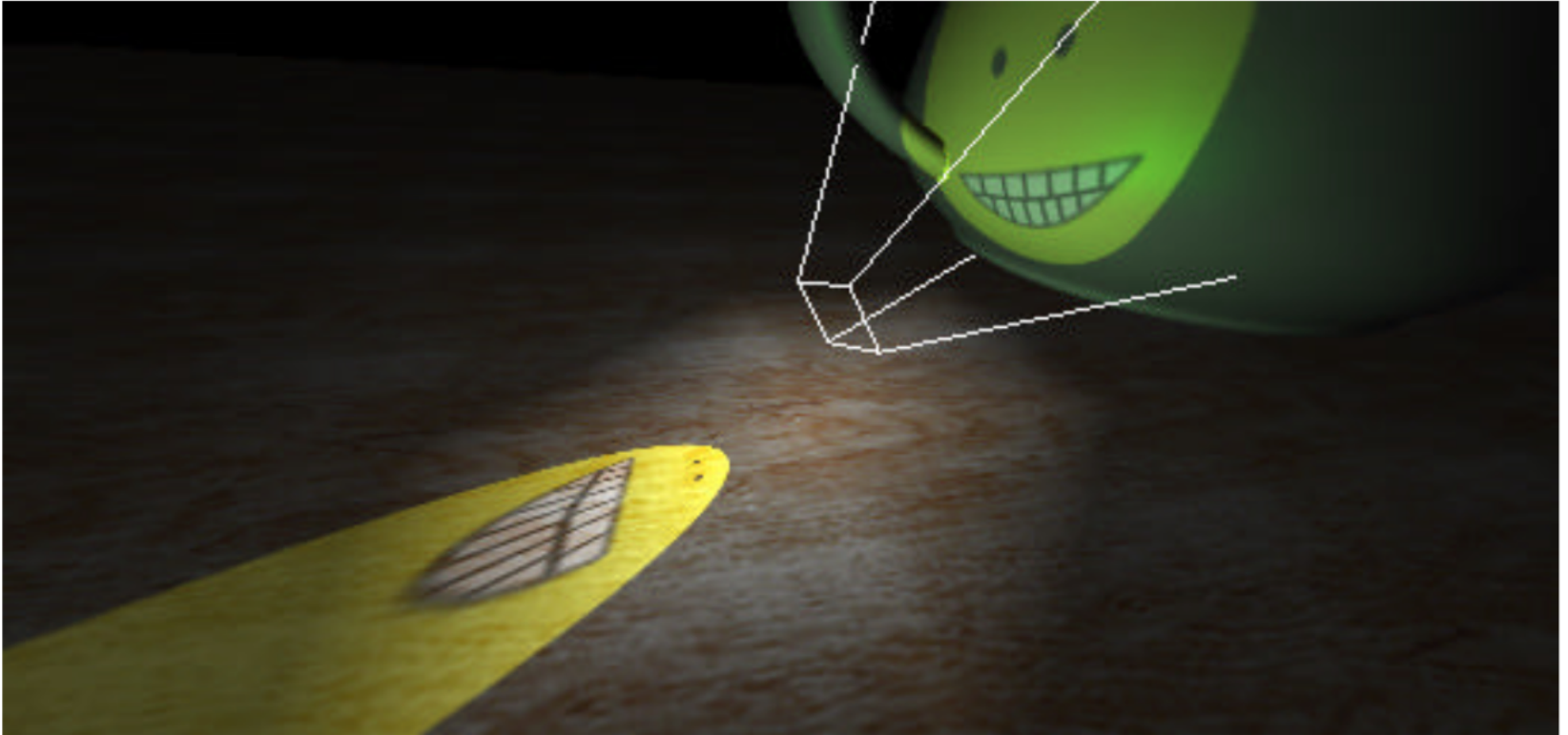


```
glMatrixMode(GL_TEXTURE);  
glLoadIdentity();  
glTranslatef(0.5,0.5,0.5);
```

**Much simpler in newer OpenGL
(will discuss later)**

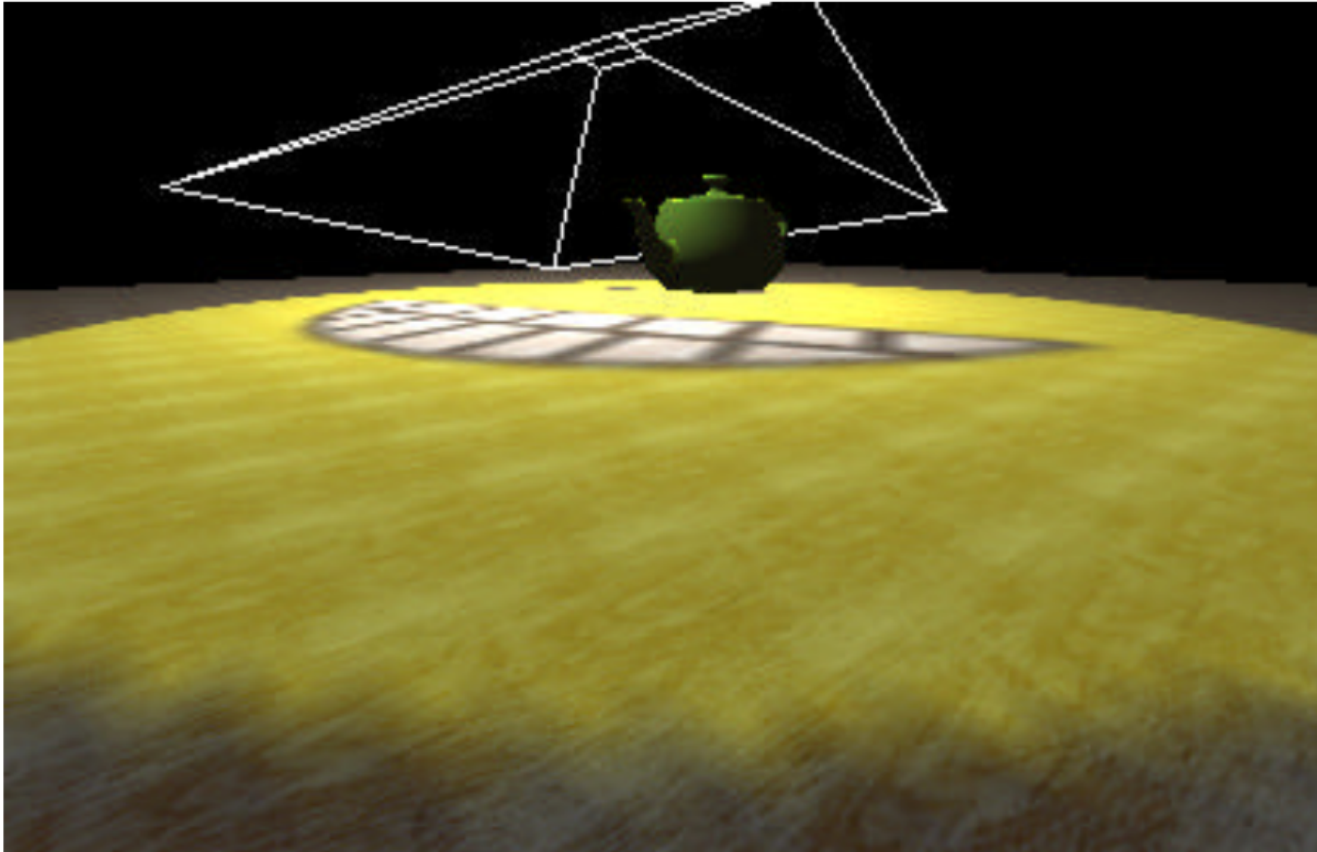
```
glMatrixMode(GL_MODELVIEW);  
.  
.  
glMatrixMode(GL_MODELVIEW);
```

Projective Texturing: Issues



$$q'' < 0$$

Projective Texturing: Issues



Severe Aliasing



Textures



CS 148: Summer 2016
Introduction of Graphics and Imaging
Zahid Hossain